[Alexander and Sadiku, 2009, Q4.6]: Linearity

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Method 1:

From experiment 1, $V_s^{(1)} = 12 \rightarrow V_o^{(1)} = 4$

For experiment 2, $V_o^{(2)}$ becomes $3 \times V_o^{(1)}$

same factor

So, $V_s^{(2)}$ should be $3 \times V_s^{(1)} = \boxed{48\ \text{V}}$

For experiment 3, $V_s^{(3)}$ becomes $\frac{1}{12} V_s^{(1)}$

same factor

So, $V_o^{(3)}$ should be $\frac{1}{12} \times V_o^{(1)} = \frac{1}{12} \times 4 = \boxed{\frac{1}{3}\ \text{V}}$

For experiment 4, $V_o^{(4)}$ becomes $-\frac{1}{2} V_o^{(1)}$

same factor

So, $V_s^{(4)}$ should be $-\frac{1}{2} V_s^{(1)} = -\frac{1}{2} \times 12 = \boxed{-6\ \text{V}}$

Method 2:

Consider two experiments on the same linear circuit:

Input Output

$$V_s^{(1)} \rightarrow \boxed{\text{Linear Circuit}} \rightarrow V_o^{(1)}$$

$$V_s^{(2)} \rightarrow \boxed{\text{Linear Circuit}} \rightarrow V_o^{(2)}$$

If we write $V_s^{(2)} = \frac{V_s^{(2)}}{V_s^{(1)}} \times V_s^{(1)}$,

then $V_o^{(2)}$ should be

use same factor

$$\frac{V_s^{(2)}}{V_s^{(1)}} \times V_o^{(1)}$$

By similar reasoning.

Hence, $\frac{V_o^{(2)}}{V_s^{(2)}} = \frac{V_o^{(1)}}{V_s^{(1)}} = \frac{V_o^{(3)}}{V_s^{(3)}} = \frac{V_o^{(4)}}{V_s^{(4)}}$

In other words, $\frac{V_o}{V_s}$ should be constant for all experiments.

From experiment 1, we have $\frac{V_o}{V_s} = \frac{4}{12} = \frac{1}{3}$.

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For experiment 2, we should have $\frac{V_o}{V_s} = \frac{16}{V_s} = \frac{1}{3} \Rightarrow V_s = 3 \times 16 = \boxed{48\ V}$

For experiment 3, we should have $\frac{V_o}{V_s} = \frac{V_o}{1} = \frac{1}{3} \Rightarrow V_o = \boxed{\frac{1}{3}\ V}$

For experiment 4, we should have $\frac{V_o}{V_s} = \frac{-2}{V_s} = \frac{1}{3} \Rightarrow V_s = -2 \times 3 = \boxed{-6\ V}$

[Alexander and Sadiku, 2009, Q4.8]: Superposition; Two Voltage Sources

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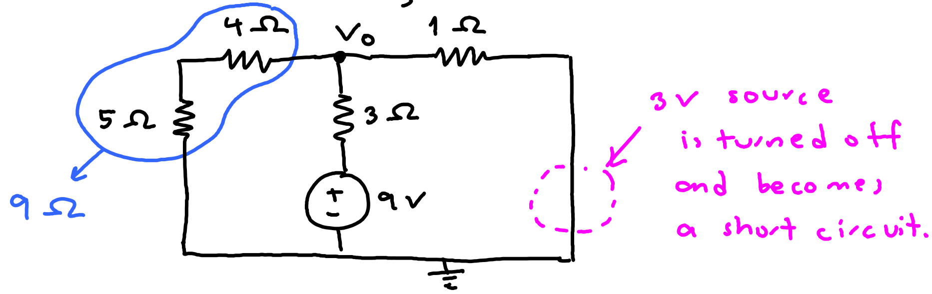
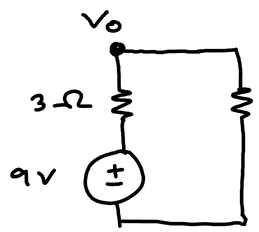
We will activate one source at a time

a) When the 9 V is on, the circuit becomes



Note that the 9 Ω and 1 Ω are in parallel. Hence, we have

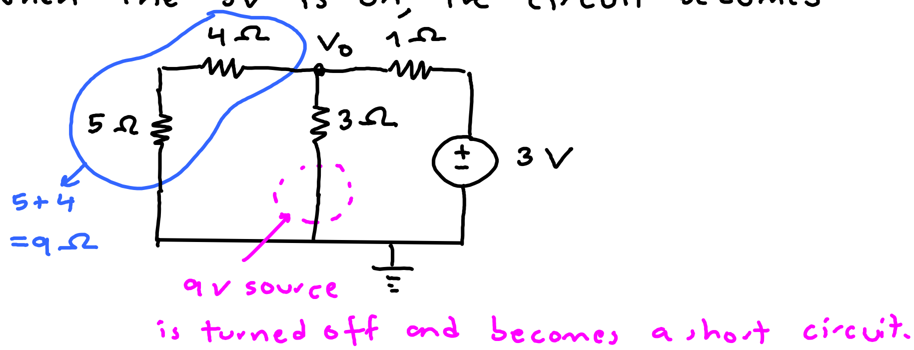


$$9 // 1 = \frac{9 \times 1}{9+1} = \frac{9}{10} = 0.9$$

By the voltage divider formula, $V_o = \frac{0.9}{3+0.9} \times 9 = \frac{\cancel{0.9}^{3}}{\cancel{3.9}_{13}} \times 9 = \frac{27}{13}$ V

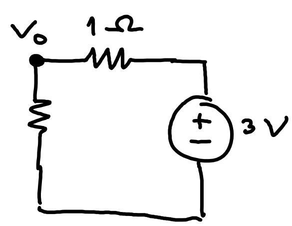
Refer to this by $V_o^{(a)}$

b) When the 3V is on, the circuit becomes



Note that the 9 Ω and 3 Ω are in parallel. Hence, we have

$$\frac{9}{4} = \frac{9 \times \cancel{3}^{1}}{\cancel{9}_{3} + \cancel{3}_{1}} = 9 // 3$$



By the voltage divider formula, $V_o = \frac{\frac{9}{4}}{9} \times 3 = \frac{9}{9+4} \times 3 = \frac{27}{13}$ V

By the voltage divider formula, $V_o = \dfrac{\frac{9}{4}}{\frac{9}{4}+1} \times 3 = \dfrac{9}{9+4} \times 3 = \dfrac{27}{13}\,V$

Refer to this by $V_o^{(b)}$.

c) By superposition theorem, $V_o = V_o^{(a)} + V_o^{(b)} = \dfrac{27}{13} + \dfrac{27}{13} = \dfrac{54}{13}\,V \approx 4.154\ V$

[Alexander and Sadiku, 2009, Q4.12]: Superposition; Two Voltage Sources and One Current Source

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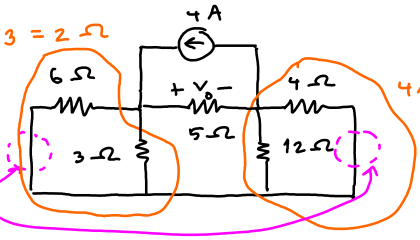
a) When only 4 A is on:

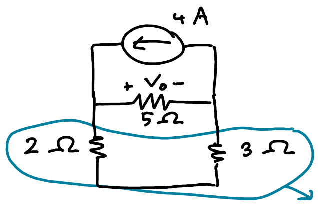


$6 // 3 = 2\,\Omega$

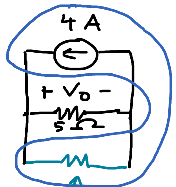
$4 // 12 = \frac{12 \times 4}{12 + 4} = \frac{\cancel{12}^{3} \times \cancel{4}}{\cancel{16}_{\cancel{4}}} = 3\,\Omega$

Voltage sources are turned off and replace by short circuits

$\equiv$



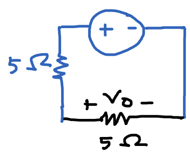
$\equiv$



$2 + 3 = 5\,\Omega$

$4 \times 5 = 20\ V$

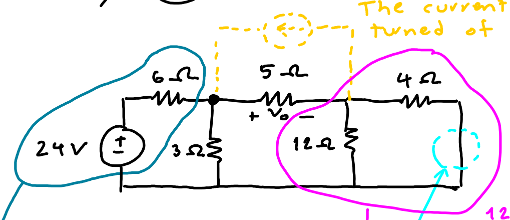
$\equiv$



voltage divider

$V_o = \frac{5}{5+5} \times 20 = 10\ V$

b) When only 24 V is on,

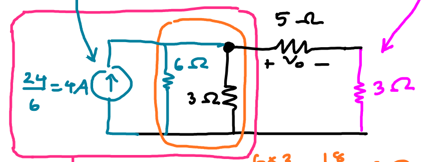


The current source is turned off and replaced by open circuit.

$12 // 4 = 3\,\Omega$

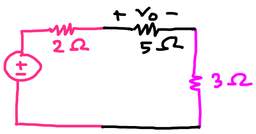
The voltage source is turned off and replaced by short circuit.

Source transformation



$\frac{24}{6} = 4A$

$6 // 3 = \frac{6 \times 3}{6+3} = \frac{18}{9} = 2\,\Omega$

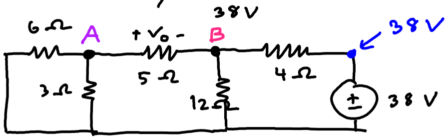


$4 \times 2 = 8\ V$

voltage divider

$V_o = \frac{5}{2+5+3} \times 8 = \frac{40}{10} = 4\ V$

c) When only 38 V is on,



Here, let's try nodal analysis:

KCL @ A: $\frac{V_A}{6} + \frac{V_A}{3} + \frac{V_A - V_B}{5} = 0$

KCL @ B: $\frac{V_B - V_A}{5} + \frac{V_B}{12} + \frac{V_B - 38}{4} = 0$

$\Rightarrow V_A = \frac{57}{10}, \quad V_B = \frac{399}{20}$

$\Rightarrow V_o = V_A - V_B = \frac{57}{10} - \frac{399}{20} = -\frac{57}{4}\ V \approx -14.25\ V$

d) By superposition theorem,

$V_o = 10 + 4 + (-14.25) = -0.25\ V = \boxed{-250\ mV}$

[Alexander and Sadiku, 2009, Q4.27]: Source Transformation; Two Voltage Sources and One Current Source

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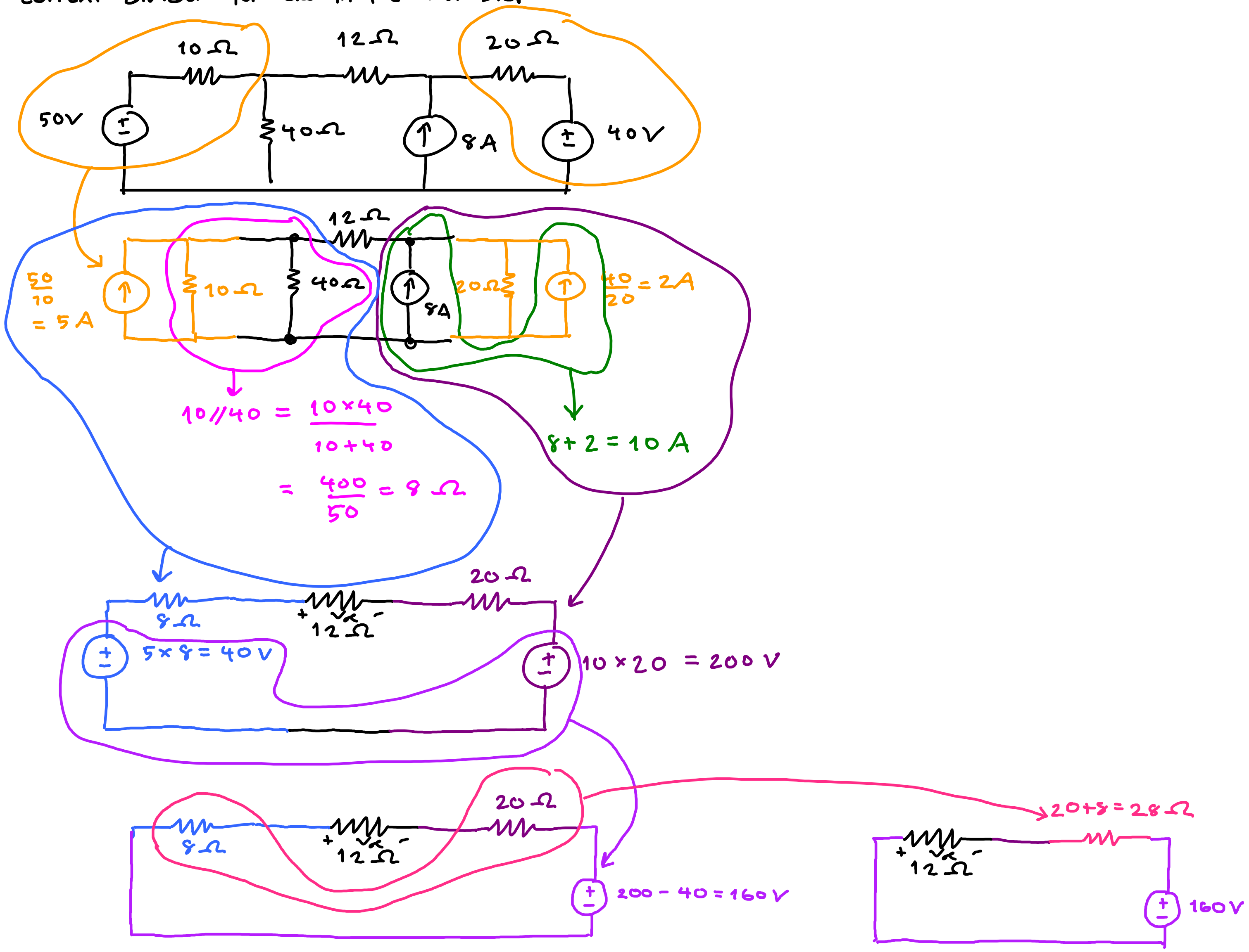
In this class, when possible, we will try to solve any problem that requires the use of source transformation by successive applications of

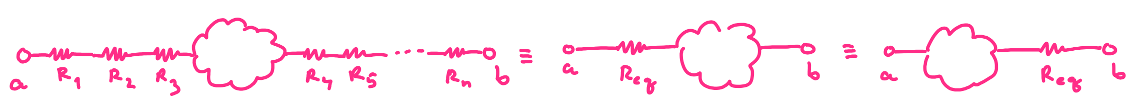
1) source transformations

2) resistor combination(s) and/or source combination(s)

The voltage and/or the current of interest can be found by applying the voltage or current divider formula in the last step.



To see why we can combine the 8 Ω and 20 Ω, recall our generalization of the combination of resistors in series:



$$R_{eq} = \sum_{i=1}^{n} R_i$$

Here, terminals a-b are across the 12 Ω. The 160 V voltage source is the

$R_{eq} = \sum_{i=1} R_n$

Here, terminals a-b are across the 12 Ω. The 160 V voltage source is the in the generalization above. Therefore, although the 8 Ω and 20 Ω are not connected directly in series, they are "in series" from the point of view of terminals a-b.

Finally, by the voltage divider formula, $V_x = -\frac{12}{12+28} \times 160 = \boxed{-48 \text{ V}}$

[Alexander and Sadiku, 2009, Q4.22]: Source Transformation

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In this class, when possible, we will try to solve any problem that requires the use of source transformation by successive applications of

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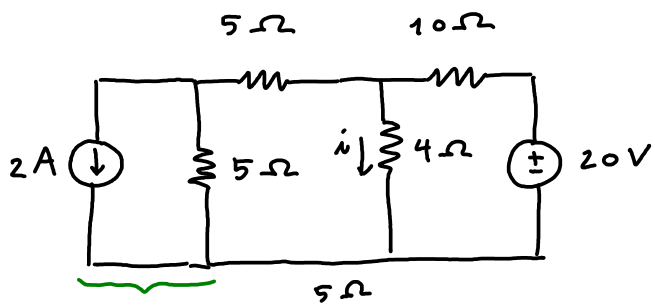

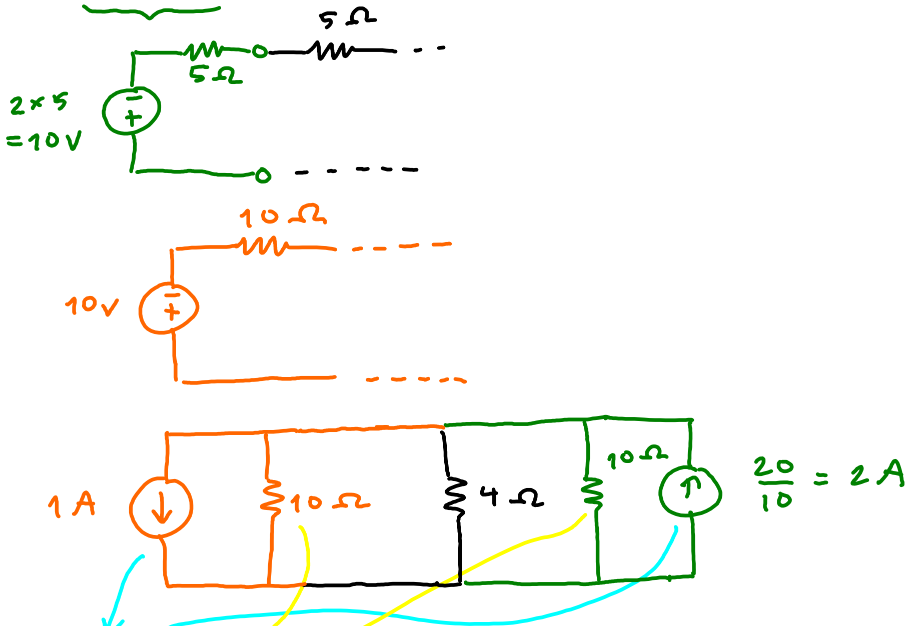

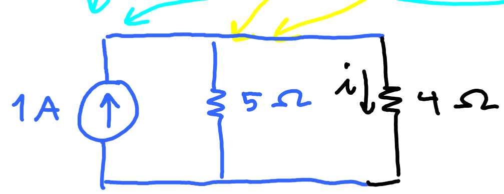

$$i = \frac{\frac{1}{4}}{\frac{1}{4}+\frac{1}{5}} \times 1 = \boxed{\frac{5}{9} A}$$

current divider formula